Plume-Rise Effect on Natural Convection Heat Transfer in Staggered Arrays of Circular Heating Elements

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The interactive effect of warm wakes induced by lower positioned heating elements on the natural convection of upper heating elements has been experimentally studied using a Mach-Zehnder interferometer. The heating elements consisting of circular horizontal tubes are arrayed in a staggered configuration. It is found that the qualitative behavior of heat transfer around each heating element is to some degree invariant with the tube spacing ratio and tube elevation in the tested range. Based on typical characteristic length, which includes tube diameter, tube spacing, and type of tube bundle arrangement, generalized correlating equations for average Nusselt numbers on all tubes are presented in the modified Grashof number range of 1×10^5 to 2×10^8. The staggered arrangement of heating elements has always had a better heat-transfer performance than the in-line arrangement when other experimental conditions are maintained the same. However, the degree of enhancement depends on the tube spacing ratio.

Nomenclature

D = tube diameter, m
Gr = Grashof number = $g\beta(T_w - T_\infty)\xi^3/\nu^2$
Gr^* = modified Grashof number = $g\beta q_w''' \xi^4/k_m \nu^2$
g = gravitational acceleration, m/s
h = local heat-transfer coefficient around a tube circumference, W/m²K
$\bar{h}$ = average heat-transfer coefficient, w/m² K
k = thermal conductivity, w/mK
Nu_θ = local Nusselt number $= h\xi/k_m$
$\overline{Nu}$ = average Nusselt number on a tube $= (1/\bar{h})\xi/k_m$
Nu_θ^* = normalized local Nusselt number $= Nu_\theta/\overline{Nu}$
$\overline{Nu_i^*}$ = ratio of the average Nusselt number of the ith tube to the average Nusselt number of the bottom tube $= \overline{Nu_i}/\overline{Nu_1}$
n = number of horizontal rows of tubes
P = center-to-center spacing between tubes, m
q'' = heat flux, w/m²
r = radial distance from the tube center, m
S = spacing ratio to the tube diameter $= P/D$
T = temperature, K
α = thermal diffusivity, m²/s
β = coefficient of volumetric expansion, K⁻¹
ξ = characteristic length defined by Eqs. (8) and (9), m
θ = counterclockwise circumferential angle along the tube perimeter form the downward vertical line, deg
ϕ = tube bundle inclination angle from the vertical line, deg
ν = kinematic viscosity, m²/s

Subscripts

i = tube identification with respect to bottom tube, 1 = bottom tube
m = mean value at $(T_w + T_\infty)/2$, K
w = condition on the tube surface
∞ = at ambient condition

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Introduction

THE interaction of natural convective plumes arising from lower heating elements with upper heating sources is of great importance to many engineering systems. This is especially true to the designers of electronic cooling systems. Increase of heat dissipation through electronic components enhances component durability and allows more compact packaging of the devices.

The natural convection heat transfer from multiheating elements is significantly affected by the degree of plume-rise effect. The effect, in turn, depends on the geometrical configuration of heating elements such as element shape, element size, spacing between elements, and arrangement of heating elements (i.e., in-line or staggered). Although there have been many studies on the interaction of natural convection heat transfer for the in-line arrays,[1,6] those for the staggered arrays[7-9] are limited.

In one of the earliest studies, Eckert and Soehngen[7] have investigated the heat-transfer characteristics from a staggered array of three heating elements. They found that heat transfer is much enhanced compared with the case of the in-line arrangement. Tilman[8] and Tsubouchi and Saito[9] have studied the natural convection heat transfer in banks of uniformly heated horizontal cylinders. They also observed that the total average heat transfer for the staggered array is about 17% higher than that for the in-line array. The reason for the enhancement is that the wake in the staggered arrangement is more cooled than that in the in-line arrangement primarily due to the thermal diffusion before impacting the upper heating element. Recently, Farouk and Güceri[10] have numerically investigated the natural convection heat transfer from double rows of closely spaced heating elements for both in-line and staggered arrays. Their study revealed the trend in heat transfer on the upper tube, which is contradictory to the previous experimental results.[7-9] The heat transfer on the upper tubes in the in-line array was higher than that in the staggered array.

Although overall heat-transfer rates in the staggered array have been studied to some degree, little information exists on the local heat-transfer coefficient as well as its qualitative behavior along the heating elements in staggered arrangements. Adequate empirical correlations for the average Nusselt numbers on circular heating elements are not available over a certain range of test variables.

In the present study, local as well as average heat-transfer

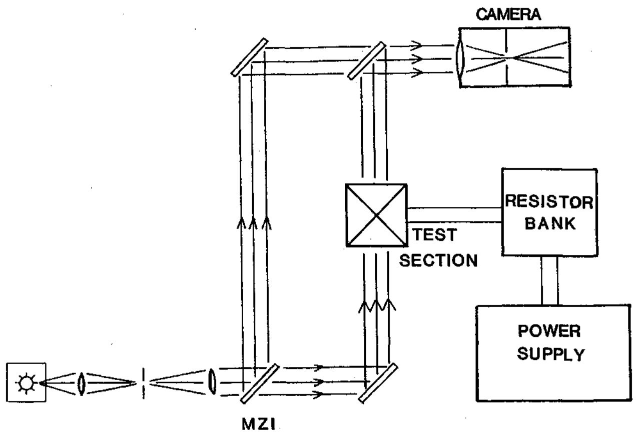

Fig. 1 Schematic of experimental apparatus.

coefficients along the hot tube were obtained through interferometric techniques. The tested center-to-center tube spacing ratios to diameter were 2 and 4, respectively. The offset angles between staggered tubes were 30 and 45 deg, respectively. The local values were integrated over the heating element surface to determine the average heat transfer. Generalized correlating equations for the average Nusselt numbers on each tube which are applicable for both in-line and staggered arrays were developed.

Test Procedure and Analysis

The experimental setup consisted of a Mach-Zehnder interferometer, a tube bundle, a tube positioner, and a power supply system as shown schematically in Fig. 1. The details of the Mach-Zehnder interferometer (MZI) is presented in the literature.[11] Consequently, we will describe only a brief summary on the other apparatus and test procedure.

In this study, two tube bundles of 3.2 and 6.4 mm, respectively, in tube diameter were fabricated. Each tube bundle was made of thin-walled-type 304 stainless steel hollow circular tubes arranged in square array with 1.25-cm center-to-center spacing between tubes. The configuration of the tube bundles was maintained using 1.5-mm brass rods welded to the tubes in the front and back planes so that the columns of tubes were connected in series and the rows of tubes in parallel. The tubes were electrically heated uniformly by a dc current passing through the thin tube walls. One tube bundle consisting of 3.2-mm-diam tubes was 6.25 cm high with five rows. The other consisting of 6.4-mm-diam tubes was 8.75 cm high with seven rows. The tube bundles were 15 cm wide and 10.2 cm long. The low resistance of the tube bundles (less than 0.1 Ω) necessitates the use of a resistor bank to control the power input so that a desirable current through a tube bundle could be obtained.

A positioner 60 cm high, 25 cm long, and 35 cm wide was used to hold the tube bundle in the air. The top plate of the positioner was designed to keep the test tube bundle tilted at any angle from/to the vertical. In this manner a staggered array was able to be arranged as shown in Fig. 2. The interferometer light beam was small (3.5 cm) compared with the test-section height. Hence, the positioner was mounted on a laboratory jack. The vertical movement of the positioner allowed one to view an entire vertical array of the tube bundle.

In experiments, first, a tube bundle was carefully hung to the positioner at a certain inclination angle (i.e., $\phi = 30$ or 45 deg) with respect to the horizontal. The alignment was checked with a protractor and level instrument. The tube bundle was then aligned in parallel to the light beam by keeping a shadow of the tube cross section on the objection screen of the MZI as sharp and circular as possible. After the test-section alignment, the MZI was then adjusted to obtain an infinite fringe pattern by moving one of the interferometer mirrors.

Before a test, ambient conditions including temperature,

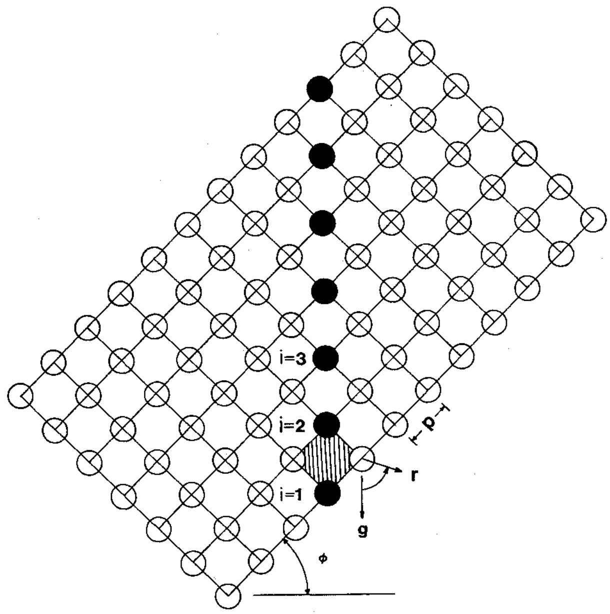

Fig. 2 Schematic of tube bundle geometry showing nomenclature.

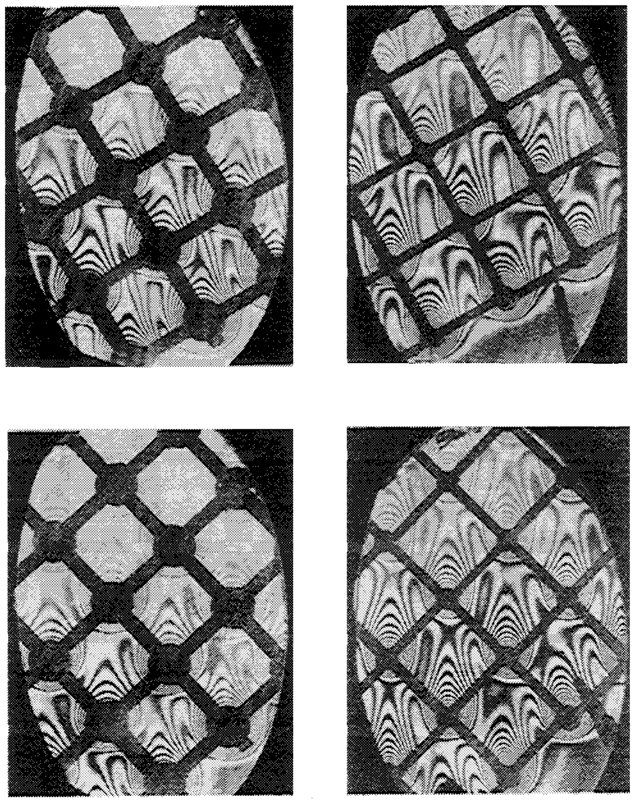

Fig. 3 Interferograms in staggered tube bundle for different experimental conditions.

pressure, and relative humidity were recorded. After a proper length of time for a stable condition in the test section, the interferograms were taken with 10.2 × 12.7 cm Kodak Tri-X film at $\lambda = 4370$ Å. The exposure time was 1/10 s. All the tests were conducted in a draft-free zone.

Figure 3 presents interferograms typical of those obtained in this study. The dark and bright contours are isotherms. Measurements of local temperature gradients were obtained from the corresponding interferograms and used to determine the local heat-transfer coefficients. The local Nusselt number was based upon the characteristic length ξ and the total tempera-

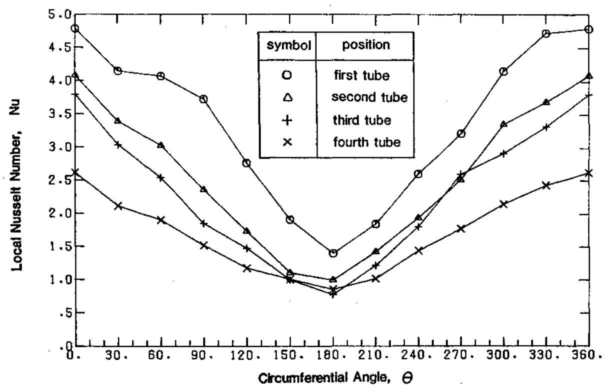

Fig. 4 Local Nusselt number variation with tube circumferential angle, $S = 2.0$, $\phi = 45$ deg, and $Gr^* = 2.7 \times 10^3$.

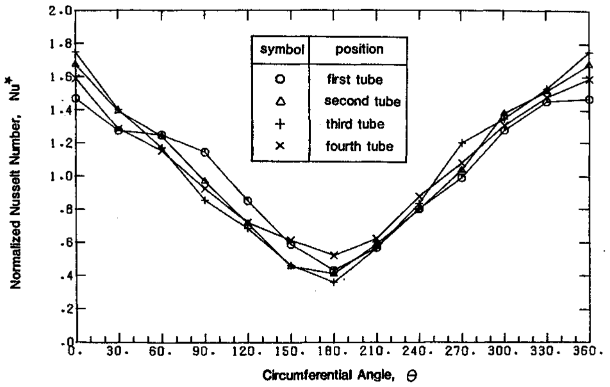

Fig. 6 Normalized Nusselt number variation with tube circumferential angle, $S = 2.0$, $\phi = 30$ deg, and $Gr^* = 2.7 \times 10^3$.

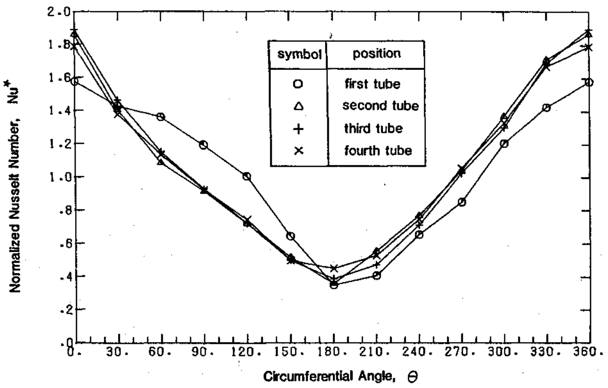

Fig. 5. Normalized Nusselt number variation with tube circumferential angle, $S = 2.0$, $\phi = 45$ deg, and $Gr^* = 4.2 \times 10^3$.

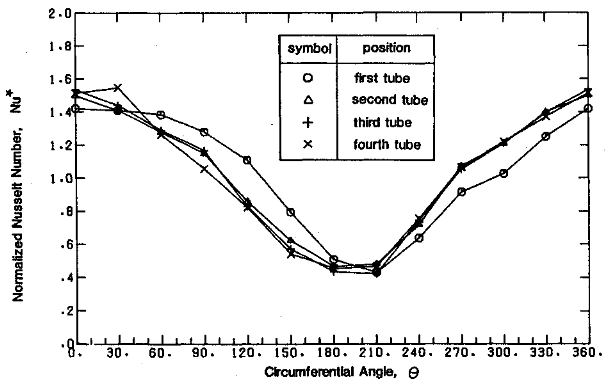

Fig. 7 Normalized Nusselt number variation with tube circumferential angle, $S = 4.0$, $\phi = 45$ deg, and $Gr^* = 0.4 \times 10^3$.

ture difference between the tube surface and the ambient air $T_w - T_\infty$. The choice of the characteristic length is arbitrary. In most cases of a single circular tube, the tube diameter was used for the characteristic length. Here, we initially used the tube diameter but later a different characteristic length. The local Nusselt number was calculated from the nondimensional temperature gradient at the tube surface, which is then multiplied by the ratio of the thermal conductivity evaluated at the surface temperature to that evaluated at the mean temperature as follows:

$$Nu_\theta = \frac{h_\xi}{k_m} = -\frac{k_w}{k_m}\frac{\partial[(T-T_\infty)/(T_w-T_\infty)]}{\partial(r/\xi)}\bigg|_{r=\frac{D}{2}} \qquad (1)$$

where k_m is evaluated at the mean temperature $(T_w + /T_\infty)/2$. The average Nusselt number was found by numerically integrating the local values over the entire tube surface, i.e.,

$$\overline{Nu} = \frac{\bar{h}\xi}{k_m} = \frac{1}{2\pi}\int_0^{2\pi} Nu_\theta \, \mathrm{d}\theta \qquad (2)$$

The corresponding value of the local Grashof number was based on properties also evaluated at the arithmetic mean temperature. Since the tests were conducted at the condition of a constant heat flux from each tube, the modified Grashof number was used through this study. The modified Grashof number was defined as follows

$$Gr^* = Gr \times Nu_\theta = \frac{g\beta q_w'' \xi^4}{k_m \nu^2}$$

Results and Discussion

Interferograms have been analyzed along the tube array marked by a solid circle as shown in Fig. 2. By using a tube diameter for a characteristic length, local Nusselt numbers at the surface of each tube have been calculated. The local heat-transfer variations with the tube circumferential angle θ at different tube elevations are shown in Fig. 4. The result is for the tube bundle of $S = 2.0$, $\phi = 45$ deg, and $Gr^* = 2.7 \times 10^3$. As expected, the heat transfer generally decreases as the tube elevation increases due to the effect of the warm plumes induced by the lower heating elements. The local heat transfer around the tube perimeter reaches the maximum, at $\theta = 0$ deg regardless of the tube elevation.

In order to generalize the local heat transfer behavior around a tube circumference, the local Nusselt numbers are normalized with respect to the average Nusselt number of the concerning tube as follows:

$$Nu_\theta^* = \frac{Nu_\theta}{\overline{Nu}} \qquad (3)$$

where $\overline{Nu}$ is the average value calculated in Eq. (2). Since this is simply the ratio of the local value to the average, the normalized value becomes independent of the choice of the characteristic length. The typical local normalized Nusselt number variations for the suggested tube bundles of $S = 2.0$ and $\phi = 45$ deg, $S = 2.0$ and $\phi = 30$, and $S = 4.0$ and $\phi = 45$ deg are shown in Figs. 5-7, respectively. From these figures, it is evident that the inclination angle ϕ tested in this research does not considerably affect the qualitative behavior of heat transfer. It is also noteworthy that for closely spaced tube bundles such as $S = 2.0$ and 4.0, the qualitative trend of heat transfer is generally similar regardless of the heating element location, i.e., on the bottom tube and on the upper tubes. Here, the upper tubes are under the influences of warm wakes. This observed behavior is quite different from that for the in-line tube bundles as indicated in the literature.[11] For the closely spaced in-line tube bundles, the heat-transfer behavior of the upper tubes is significantly different from that of the bottom tubes. For a staggered array, however, any significant difference in the qualitative heat-transfer behavior between two regions is not observed. Therefore, the heat-transfer behavior of the upper tubes is somehow similar to that of a bottom tube which obtains the maximum heat transfer at $\theta = 0$ deg and the minimum value at $\theta = 180$ deg, respectively. This phenomenon may be explained by the fact that for the staggered tube bundles, the influence of upcoming warm plume induced by the lower heating elements prevails all around the concerned upper tube so that the upper heating element is situated in what appears to be single heating element in a forced convective flow to some degree.

Since the profiles of the normalized local Nusselt numbers in a tube bundle are considerably uniform for all tubes as shown in Figs. 5–7, a generalized profile for the tube bundle can be obtained by finding average values at individual circumferential angle θ. The deviations of the normalized local Nusselt numbers from the generalized values are within $\pm 5\%$. The generalized profiles for the other cases are plotted in Fig. 8. It is interesting to note that the trend in the variation of the normalized local Nusselt numbers is in general independent of the tube spacing ratio and the heat flux in the tested range.

Average Nusselt numbers based on the tube diameter are also considered to investigate the quantitative heat transfer in the staggered tube bundles. In order to observe the variation of the average heat-transfer coefficient with the tube elevation, the average Nusselt number of each tube is normalized with respect to that of the bottom tube in a tube bundle. The profiles for the tube bundles of $S = 4.0$ and $S = 2.0$ are, respectively, plotted in Fig. 9. The results for the corresponding in-line array, which were reported in the previous literature,[11] are also presented for comparison. Since the profile was not much dependent on the heat flux, i.e., three different G_r^* for the tube bundle of $S = 4.0$ and two different G_r^* for $S = 2.0$, their average value of these is used for the plot. The deviations of the individual data points from the average are within $\pm 4\%$. From Fig. 9, it can be observed that the average Nusselt numbers at the second, third, and fourth tubes for $S = 4.0$ are 82, 75, and 69%, respectively, of that at the bottom tube. For $S = 2.0$, the ratios are 75, 66, and 58%, respectively. Comparing results of the present staggered tube bundle with those for the in-line tube bundle, it is noted that the average values of the staggered array for $S = 4.0$ are $7 \sim 9\%$ higher, depending on the tube elevation, than those for in-line array, whereas for $S = 2$ these are about 14–18% higher. As the tube spacing ratio becomes smaller, the influence of upcoming plumes gets stronger. Accordingly, the heat transfer of the staggered arrangement increases. On the other hand, it can be expected that there exists no significant differences in heat transfer between in-line and staggered arrays as the tube spacing ratio increases. A previous report[11] indicated that for the in-line tube bundles, there is not apparent influence of the upcoming plumes on the qualitative heat transfer of the upper tubes at $S = 6$ or greater. Therefore, it is expected that there would be no significant difference in heat transfer between in-line and staggered tube bundles if the tube spacing ratio is greater than 6.

The nondimensional parameters involved in the present study, namely, Nusselt number and modified Grashof number, depend on the characteristic length ξ. When only the tube diameter is selected for the characteristic length, the correlation equations for the heat transfer may not be generalized because other geometrical configurations, such as tube spacing and type of arrangement (in line or staggered) are not included. Consequently, another characteristic length is introduced to correlate the heat-transfer results.

For the forced convection of in-line tube bundles in a cross flow, Kays and London[12] have employed a hydraulic tube diameter for the characteristic length defined as

$$\xi = D_h = \frac{4A_cP}{A_t} \tag{4}$$

where P is the longitudinal pitch of the tube bundle in parallel to the flow direction, A_c the flow cross-sectional area, and A_t the total heat-transfer area of a tube. Tilman[8] first used this characteristic length in natural convection heat transfer within a tube bundle. Based on this, he correlated the overall heat-transfer coefficients of tube bundles over the Rayleigh number range of 10^2 to 10^6 as follows:

$$Nu_{\text{over}} = 0.057(Ra)^{0.5} \tag{5}$$

for the in-line tube bundle, and

$$Nu_{\text{over}} = 0.067(Ra)^{0.5} \tag{6}$$

for the staggered arrangement, where Nu_{over} is the overall average Nusselt number of the entire tube bundle and Ra the Rayleigh number. Following a similar characteristic length as that of Tilman,[8] Choi[11] has developed correlation equations for the average heat transfer of an individual tube at different elevation in in-line bundles as follows:

$$\overline{Nu_i} = C_i(Gr^*)^{n_i} \tag{7}$$

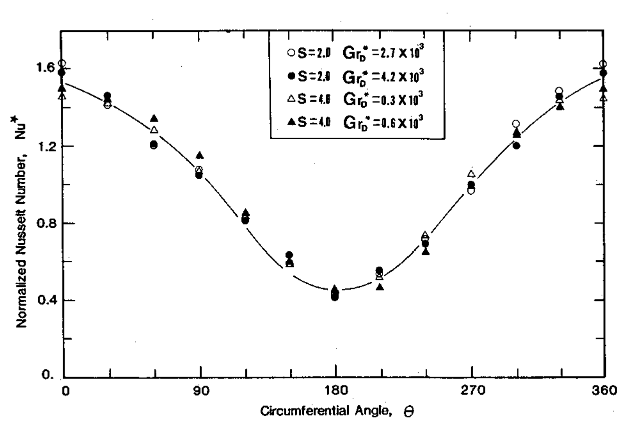

Fig. 8 Generalized profile of normalized local Nusselt numbers, $\phi = 45$ deg.

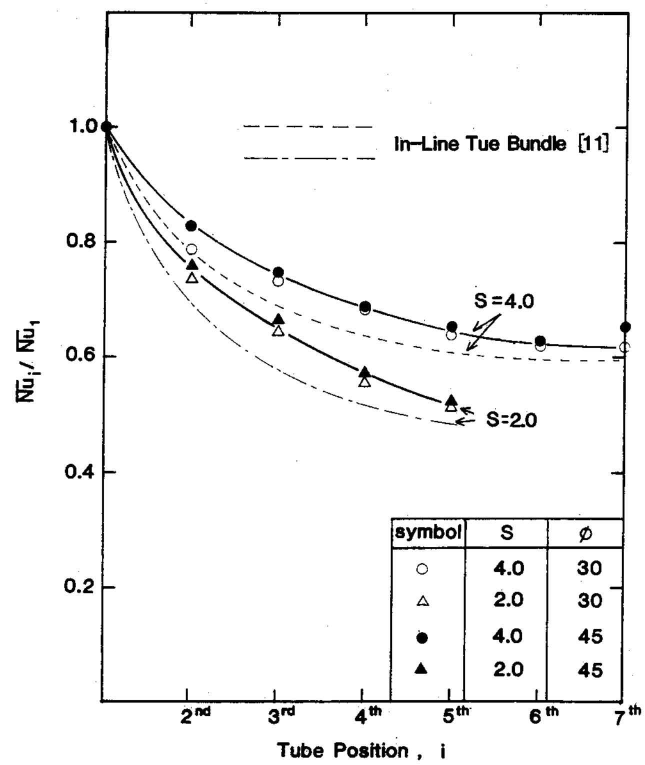

Fig. 9 Average Nusselt number variation with tube elevation.

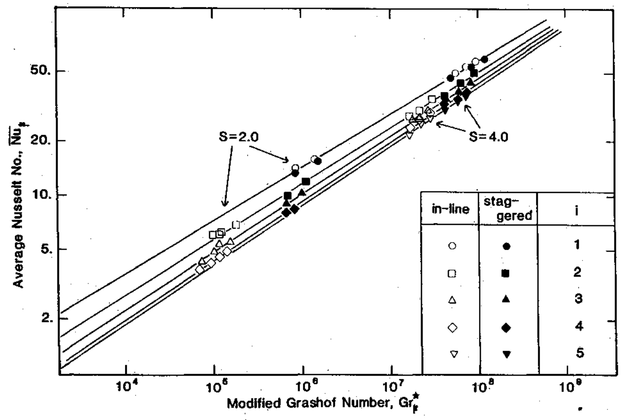

Fig. 10 Empirical correlation for average Nusselt numbers on individual tubes for both staggered and in-line tube bundles.

where C_i and n_i are constants, which depend on the tube location, and the subscript i represents the tube elevation relative to the bottom tube. The characteristic lengths for the bottom tube and the upper tubes have been defined in a different way because the qualitative heat-transfer behavior is distinctly different for both cases. For the upper tubes which are under strong influence of the upcoming warm plumes, the characteristic length adopted by Kays and London[12] was used. It is defined as follows:

$$\xi = \frac{4}{\pi}(S-1)P \qquad (8)$$

For the bottom tube, however, the characteristic length is defined as follows

$$\xi = (\frac{4}{\pi}S^2 - 1)D \qquad (9)$$

For the present staggered tube bundles, we use the same formula for the characteristic length as that given in Eq. (9). This is because, as discussed earlier, the general qualitative behavior of heat transfer along any tube in a staggered tube bundle is to some degree similar to that along the bottom tube. Therefore, for the staggered arrangement, the flow cross-sectional area A_c which appears in Eq. (4) is defined as depicted in Fig. 2. By using the characteristic length defined in Eq. (9), which is independent of the tube elevation, the average Nusselt numbers on the tubes are plotted with the modified Grashof numbers in Fig. 10. The results for the in-line tube bundles presented in the literature[11] are also presented for comparison. It is interesting to note that the data for both in-line and staggered tube bundles are represented by the same correlating equation but with different characteristic lengths. The correlation is represented by Eq. (7) and the coefficients C_i and n_i are given in Table 1. The characteristic lengths used in this study as well as those in Ref. 11 are given in Table 2. The correlations fit the data within $\pm 12\%$. The significance of the correlations is that the independent variables, such as tube spacing, tube diameter, tube bundle arrangement, and heat flux, are combined together into the modified Grashof number which is based on the characteristic length defined by Eqs. (8) and (9). From the correlations, it can be found that the degree of enhancement of heat transfer in a staggered array depends on the tube-spacing ratio. For instance, considering the characteristic lengths for both in-line and staggered arrays as given in Table 2 and the coefficients C_i and n_i as given in Table 1, it is noted that the heat-transfer rate from the staggered tube bundle of $S = 2$ is about 15% greater than that corresponding to the in-line tube bundle, whereas the enhancement becomes just 7% for $S = 4.0$.

Conclusions

The results of the experimental investigation are summarized as follows:

Table 1 Coefficients for use in Eq. (7)

i	C_i	n_i
1	0.204	0.306
2	0.148	0.316
3	0.110	0.322
4	0.089	0.336
5	0.085	0.336

Table 2 Characteristic lengths defined in Eqs. (8) and (9)

S	Staggered	In-line[11]	
2.0	2.62 cm	Bottom tube	2.62 cm
		Upper tubes	1.62 cm
4.0	6.20 cm	Bottom tube	6.20 cm
		Upper tubes	4.85 cm

1) Qualitative heat transfer in a staggered tube bundle is quite different from that in an in-line tube bundle.

2) Even though the upper tubes are under a strong influence of the upcoming plumes, the heat-transfer behavior around these tubes is very similar to that around a bottom tube.

3) The heat transfer in a staggered tube bundle is always better than that in the corresponding in-line tube bundle but depends on the tube-spacing ratio. For example, the heat-transfer enhancements are 15 and 7%, respectively, for S = 2.0 and 4.0.

4) The average Nusselt numbers are correlated with the modified Grashof number in the range of 9×10^5 to 2×10^8 as follows:

$$\overline{Nu_i} = C_i(Gr^*)^{n_i} \qquad (7)$$

where C_i and n_i are coefficients given in Table. 1. This equation is also applicable to in-line tube bundles with the characteristic length defined by Eq. (8) for the upper tubes and by Eq. (9) for the bottom tube, respectively.

References

[1]Jaluria, Y,. "Interaction of Natural Convection Wakes Arising From Thermal Sources on a Vertical Surface," *ASME Journal of Heat Transfer,* Vol. 107, No. 4, Nov. 1985, pp. 883–892.

[2]Jaluria, Y. and Gebhart, B., "On the Buoyancy-Induced Flow Arising From a Heated Hemisphere," *International Journal of Heat and Mass Transfer,* Vol. 18, No. 3, 1975, pp. 415–431.

[3]Lieberman, J. and Gebhart, B., "Interactions in Natural Convection From an Array of Heated Elements, Experimental," *International Journal of Heat and Mass* Transfer, Vol. 12, No. 11, 1969, pp. 1385–1396.

[4]Fujii, T., "Theory of the Steady Laminar Natural Convection Above a Horizontal Line Source and a Point Heat Source," *International Journal of Heat and Mass Transfer*, Vol. 6, No. 7, 1963, pp. 597–606.

[5]Sparrow, E. M. and Niethammer, "Effect of Vertical Separation Distance and Cylinder-to-Cylinder Temperature Imbalance on Natural Convection for a Pair of Horizontal Cylinders," *ASME Journal of Heat Transfer,* Vol. 103, Nov. 1981, pp. 638–694.

[6]Marsters, G. F., "Arrays of Heated Horizontal Cylinders in Natural Convection," *International Journal of Heat and Mass Transfer,* Vol. 15, No. 5, 1972, pp. 921–933.

[7]Eckert, E. G. and Soehngen, E., "Studies of Heat Transfer in Laminar Free Convection with the Zehnder-Mach Interferometer," U.S. Air Force Air Material Command, Dayton, OH, TR-5747, 1948.

[8]Tilman, E. S., "Natural Convection Heat Transfer from Horizontal Tube Bundles," American Society of Mechanical Engineers Paper 76-HT-35, 1976.

[9]Tsubouchi, T. and Saito, E., "An Experimental Study of Natural Convection Heat Transfer in Banks of Uniformly Heated Horizontal Circular Cylinders," *Heat Transfer Japanese Research*, Vol. 7, No. 4, Oct.–Dec. 1978, pp. 44–58.

[10]Farouk, B. and Güceri, S. I., Natural Convection from Horizontal Cylinders in Interacting Flow Fields," *International Journal of Heat and Mass Transfer,* Vol. 26, No. 2, 1983, pp. 231–243.

[11]Choi, K. J., "Natural Convection Heat Transfer Within Horizontal Tube Bundles," Ph.D. Thesis, University of Wisconsin, Madison, WI, 1983.

[12]Kays, W. and London, A. I., *Compact Heat Exchangers,* 2nd ed., McGraw-Hill, New York, 1964, pp. 184–190.